

Can Reducing Inequality Aggravate the Pollution Stock Problem?

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Abstract

This paper looks at redistribution issues in a differential game model of international pollution control in which countries are being differentiated by some level of non-polluting endowments. We formally establish that reducing inequality by redistributing from richly-endowed countries in favor of poorly-endowed countries can affect the pollution stock in various directions, depending on differences in emission technology. With a perfect technological diffusion among richly-endowed countries and poorly-endowed countries, reducing inequality will not affect the evolution of the pollution stock. In a world composed of richly-endowed/ high-technology countries and poorly-endowed/low-technology countries, reducing inequality will decrease the pollution stock. In a world composed of richly-endowed/low-technology countries and poorly-endowed/high-technology countries, reducing inequality will increase the pollution stock. In a more heterogeneous world composed of four types of countries, it is shown that differences in emission technology alongside with the relative size of each type of countries are important factors that affect the dynamic path of the pollution stock. Environmental policy implications are discussed.

Keywords: Pollution Stock, Environmental and Climate Change, Inequality Reduction, Income, Political Economy of Distribution, Technology, Differential Games.

JEL Classification: D62, Q54, F35, I3

1 Introduction

At the Warsaw conference on climate change held in November 2013, developed countries committed to support developing countries in emission reducing and to face climate change issues.¹ The support would consist in providing developing countries with financial resource or non polluting goods. These countries could then use non polluting goods in substitution for commodities whose production is highly CO_2 emitting or forest consuming. In other words, the goal is to reduce the inequalities between developed and developing countries in terms of financial resource or non polluting goods endowments, with the intention to meet successfully the climate change challenge. However, the concern is about the real impact of that inequalities reducing on the global emission amount. Of course, bringing non polluting resource from developed countries to developing countries is likely to reduce emission and to slow down deforestation in those developing countries. But what happens in the developed countries? Indeed, it is well known that even developed nations are still facing many economic challenges. Therefore, any transfer from these countries could need a compensation. Developed countries might then increase their production and pollute more. The global impact depends on the extent to which the decrease in the recipients' emission compensates for the increase in the donors' emission.

Major environmental problems involve stock externalities. Climate change, for instance, is a stock, not a flow problem (Stavins, 2011). In addition climate change spans international frontiers. When the externality is caused by a stock rather than a flow, environmental problems are recognized to raise intertemporal tradeoffs. Therefore, environmental policy should be concerned with the dynamic impacts of policy instruments.² In this paper, we address a specific simple question: how does the pollution stock path change in response to a more equal redistribution of resource over time? There seems to be a wide acceptance that reducing income inequality among countries will lead to an improved environment quality. There is a fairly extensive empirical literature

¹http://unfccc.int/meetings/warsaw_nov_2013/meeting/7649.php

²The pollution stock problem can also be viewed as a multigenerational, international strategic resource allocation problem.

studying the question of inequality in collective action problems. However, one drawback to these studies, though, is that they are conducted by means of static economic models.³ Much of the literature tends to conclude that equality is more likely to alleviate the problem of the commons. For instance, Cornes and Sandler (1994) show that greater inequality reduces the equilibrium level of a public good.

A paper by Dayton-Johnson and Bardhan (2002) focusses on a discrete time fishery model and show that increased inequality does not tend to favor conservation in local commons dilemma such as fisheries, groundwater-based irrigation, community grazing lands, and village forests. Their paper assumes that a form of externality in which today's decisions affects another's incentives through payoffs in the following period, while the conventional externality acts through increased costs from current decisions. Bardhan et al. (2007) show that efficiency increases with greater equality within the group of contributors and non-contributors.

Our paper makes two contributions to the literature. First, our paper differs from the existing economic models in that we address the question of inequality and stock pollution in a dynamic game model setting. The relationship between inequality and environmental quality is fundamentally dynamic, and adjustment processes relating to pollution stock levels are clearly not instantaneous. To the best of our knowledge, capturing features related to inequality and pollution stock externalities in a dynamic game model has not been done in the environmental economic literature. Second, this paper's results contrast with this stream of literature by showing that, under certain conditions, inequality reduction can raise the stock of pollution, and then aggravating the stock externality problem. This paper is the first theoretical paper that analyzes such an issue in a continuous time dynamic setting dealing with transboundary pollution. We use a modified version of the differential game of international pollution control à la Dockner and Long (1993) in which countries are differentiated by a parameter that captures an endowment that yields a pollution-free flow of benefits over time. We mainly consider two types of countries. The countries that have higher quantity of a non polluting good (richly-endowed), and the countries with lower quan-

³There are no state variables that evolve over time.

tity of the polluting good (poorly-endowed). In addition to the non polluting good, each country produces and consumes another polluting good with a given technology. Emissions strategies of poorly-endowed countries and richly-endowed countries are derived and compared. We formally established that a reduction in inequality by redistributing from the richly-endowed in favor of the poorly-endowed may lead to greater accumulation of the stock of pollution. The result depends mainly on the technology used by each country. If the more equal redistribution is in favor of the countries that have the best emission technology then reducing inequality leads to emission decrease. But, if the redistribution is in favor of the countries with lower technology then lower inequality leads to higher emission amount

The rest of the paper is organized as follows. In Section 2, we present the main features of the differential game of international pollution control. Section 3 compares the emissions strategies and pollution stocks in a world of richly-endowed with a world of poorly-endowed countries. We also consider a world composed of both types of countries and we show that, under certain conditions, a reduction in inequality by redistributing from the richly-endowed in favor of the poorly-endowed will lead to a greater accumulation of the stock of pollution. We do the same analysis in Section 4 by taking into account the fact that the richly-endowed country could have the lower technology, and the poorly-endowed country could have the higher technology. Section 5 offers concluding comments.

2 The Economic model

Consider a N -player (country) differential game played over the time interval $[0, \infty]$, in which agents are involved in a pollution-generating economic activity. Denote by $Q_i(t)$ the industrial production of country $i \in \{1, \dots, N\}$ at time t . The production gives rise to a byproduct externality, namely (gross) emissions $e_i(t)$ at time t . For simplicity it will be assumed that one unit of production gives rise to $\frac{1}{\phi_i}$ units of pollution. In other words, production is proportional to gross emissions as follows

$$Q_i(t) = \phi_i e_i(t) \tag{1}$$

The parameter $\phi_i > 0$ characterizes the emission technology in country i . A higher level of $\phi_i > 0$ means that the production gives rise to lower level of emissions. Technological change, in this sense, represents the impact of exogenous forces. It rather pertains to innovation fallout that provides the country with pollution reducing facilities (emission reducing technologies, electric car, solar energy, etc.). It is also assumed that emissions of pollution accumulate into a stock, $S(t)$ according to the transition equation:

$$\dot{S}(t) = \sum_{j=1}^N e_j(t) - kS(t), \quad \text{given,} \quad (2)$$

where $S(0) = S_0 > 0$ is the initial stock of pollution, the parameter $0 < k < 1$ is the rate at which the stock of pollution decays naturally. Equation (2) tell us that the rate of change of the carbon stock is determined by the flow of emissions and the natural degradation or dissipation of the existing stock.⁴

Each country is assumed to suffer equally from the global stock of pollution.⁵ The pollution stock S is generating instantaneous damages $D(S(t))$ at time. The damage function is assumed to be a nonlinear increasing and convex function of the stock, more specifically a quadratic:

$$D(S(t)) = \frac{b}{2}S(t)^2 \quad (3)$$

with b is a strictly positive parameter which relates to vulnerability to the stock externality problem.⁶ Transboundary environmental damage is mainly related to the accumulation of pollution, rather than the flow of emissions.

Let $u(c)$ be the instantaneous gross surplus resulting from the consumption of the services provided by c . Since we have assumed a monotone relationship between industrial production and emissions, we express the instantaneous social welfare of country i at time t for any given n -tuple emission strategy (e_1, \dots, e_N) as follows:

⁴The parameter k reflects among others the natural removal of carbon dioxide by terrestrial sinks and oceanic sinks.

⁵For this exposition, we omit regional differences in terms of vulnerability to the stock externality problem.

⁶The marginal damage is negligible if the stock of pollution is zero. For this exposition, we omit regional differences in terms of vulnerability.

$$u \left(\gamma \phi_i e_i(t) + (1 - \gamma) \left[\phi_i e_i(t) - \frac{1}{N-1} \sum_{j \neq i} \phi_j e_j(t) \right] + \pi_i \right) - D(S(t)) \quad (4)$$

where the instantaneous utility function $u(\cdot)$ is strictly increasing, concave, twice continuously differentiable with derivatives $u_c(\cdot)$ and $u_{cc}(\cdot)$, and satisfies the following Inada type assumption

$$\lim_{c \rightarrow 0} u_c(c) = \infty, \text{ and } \lim_{c \rightarrow \infty} u_c(c) = 0. \quad (5)$$

The parameter γ captures the status seeking in the behavior of the country. Each country cares about not only its own absolute production but also its relative production relative to that of others. The relative production is the difference between the absolute production $\phi_i e_i(t)$ and the average production $\frac{1}{N-1} \sum_{j \neq i} \phi_j e_j(t)$.⁷

In addition to the polluting good, each country derives utility from the consumption of a non-polluting good. We assume that countries are differentiated by the return, π_i , which they get on some initial endowment. The endowment yields a pollution free flow of income⁸. In what follows we assume there are two types of countries: countries of type 1 enjoying (each of them) a non-polluting flow π_1 and countries of type 2 enjoying a non-polluting flow π_2 . We also assume that

$$\pi_1 < \pi_2 \quad (6)$$

Countries endowed with π_1 will be named poorly-endowed countries and countries endowed with π_2 will be named richly-endowed countries.

2.1 A country's decision program

The objective of country i with a level of non-polluting flow of benefits π_i , $i = 1, \dots, N$, is to choose a pollution control strategy $\{e_i(t) : t \geq 0\}$ (or equivalently an output strategy) that maximizes the

⁷Long and McWhinnie (2012) used the term *relative performance*. Another closely related idea is the concept of interdependence in consumption discussed by Arrow et al. (2004).

⁸The initial endowment can be think of as a composite index that includes many indicators such as financial capacity and capabilities, natural capital, etc. This return on endowment can also be thought of as being related to the country's human capital and economic, social and political institutions.

discounted stream of net benefits from consumption :

$$V_i(S_0) = \max_{e_i(t): t \geq 0} \int_0^{\infty} e^{-\beta t} \left\{ u \left(\phi_i e_i(t) - \frac{(1-\gamma)}{N-1} \sum_{j \neq i} \phi_j e_j(t) + \pi_i \right) - D(S(t)) \right\} dt$$

where β is the discount rate,⁹ and the maximization is subject (2) and to $e_i(t) \geq 0$.

The corresponding current value Hamiltonian is given by

$$\mathcal{H}_i(t) = u \left(\phi_i e_i(t) - \frac{(1-\gamma)}{N-1} \sum_{j \neq i} \phi_j e_j(t) + \pi_i \right) - D(S(t)) - \lambda_i(t) \left[\sum_{j=1}^N e_j(t) - kS(t) \right],$$

where $\lambda_i(t)$ is the shadow price associated with the pollution stock. The shadow price of carbon, also known as the social cost of carbon, is the change in the discounted value of the utility of consumption denominated in terms of current consumption per unit of additional emissions.

For simplicity, we focus on interior solutions. The first order condition, co-state equation, and state equation are:

$$\phi_i u' \left(\phi_i e_i(t) - \frac{(1-\gamma)}{N-1} \sum_{j \neq i} \phi_j e_j(t) + \pi_i \right) - \lambda_i(t) = 0 \quad (7)$$

$$\dot{\lambda}_i(t) - (\beta + k)\lambda_i(t) = D'(S(t)) \quad (8)$$

$$\dot{S}(t) = e_i(t) + \sum_{j \neq i} e_j(t) - kS(t) \quad (9)$$

and the transversality condition is

$$\lim_{t \rightarrow \infty} e^{-\beta t} \lambda_i(t) S(t) = 0 \quad (10)$$

From the differential equation related to the co-state variable in equation (8), one could see that $\lambda_i(t)$ depends neither on the endowment π_i nor on the emission technology ϕ_i , so the shadow price of the pollution stock is the same for all players, i.e., $\lambda_i(t) = \lambda(t)$, $i = 1, \dots, N$. Specifically, solving the first order linear differential equation (8) gives the shadow price of the pollution stock as

$$\lambda_i(t) = e^{(\beta+k)t} \left[\lambda_0 + \int_0^t bS(\tau) e^{-(\beta+k)\tau} d\tau \right], \quad (11)$$

⁹The discount rate, β ; is assumed to be constant and identical for all countries.

where λ_0 is the initial value of shadow price of the pollution stock, and the marginal damage at time τ is $D'(S(\tau)) = bS(\tau)$. The time path of the shadow price of the pollution stock depends upon the discount rate β , the rate of natural purification k , the initial shadow price λ_0 , and the cumulative value of the marginal damages over the time interval $[0, t]$.

3 Symmetric equilibrium: poorly-endowed world vs richly-endowed world

Consider a world composed of N identical countries. They are identical in the sense that they have the same technology parameter and are of the same type (same amount of endowment). If the countries are poorly endowed (i.e. with π_1), we say that we are in a poorly-endowed world, and if they are richly endowed (i.e. with π_2), we say that we are in a richly-endowed world. Let us denote by $e_1(t)$ the emission of a typical poorly-endowed country and by $e_2(t)$ the emission of a richly-endowed country. The parameter ϕ_i ($i = 1, 2$) stands for the emission technology parameter of the countries with endowment π_i . We assume $\pi_1 = \pi_2$.

The unique symmetric equilibrium is given by

$$e_i(t) = \frac{1}{\gamma} [u'^{-1}(\lambda(t)) - \pi_i] \quad (12)$$

It is worth noting that the strategy $e_i(t)$ is time consistent since it depends upon the costate variable $\lambda(t)$. And as equation (11) shows, the costate variable $\lambda(t)$ depends on the history of the path of the state variable S ; that is $\{S(\tau) : 0 \leq \tau \leq t\}$. Such an equilibria falls in the class of history-dependent strategies which are time-consistent.¹⁰ In others words, the shadow price of the pollution stock $\lambda(t)$ is dependent of the size and shape of the pollution stock profile.

Now let us compare the emission level of pollution between the poorly-endowed world and the richly-endowed world. Since $\pi_1 < \pi_2$, it follows that

$$e_1(t) > e_2(t) \quad (13)$$

Since the return on endowment of poorly-endowed countries π_1 is lower than the return on endowment of richly-endowed countries π_2 , global pollution in the poor-endowed world is greater

¹⁰For more details about the class of history-dependent strategies are discussed, see for instance, by EEC (1980); Dockner et al. (2000)

than the global pollution in the rich-endowed world. For a given level of utility, the richly-endowed countries do not need to pollute much because they can enjoy a greater return on their non polluting endowment than poorly-endowed countries.

The stock of pollution in the world of poorly-endowed countries is higher than the stock of pollution in world of richly-endowed countries. To see this, let us denote by $S_1(t)$ the stock of pollution in the poorly-endowed world and by $S_2(t)$ the stock of pollution in the richly-endowed countries.

Solving the first order linear differential equation (9) gives an expression of the pollution stock:

$$S_1(t) = \exp(-kt) \left[S_0 + \int_0^t \exp(k\tau) N e_1(\tau) d\tau \right] \quad (14)$$

where $e_1(\tau)$ is the emission of a poorly-endowed country at time τ , and S_0 is the initial stock of pollution. In the same way, the stock of pollution in the world of richly-endowed countries is given by:

$$S_2(t) = \exp(-kt) \left[S_0 + \int_0^t \exp(k\tau) N e_2(\tau) d\tau \right] \quad (15)$$

where $e_2(\tau)$ is the emission of a richly-endowed country at time τ , and S_0 is the initial stock of pollution.

From equations (13), (14), and (15), it follows that:

$$S_1(t) > S_2(t). \quad (16)$$

3.1 Equilibrium in a world composed of poorly-endowed and richly-endowed countries (with identical emission technology, $\phi_1 = \phi_2 = 1$)

Now let's analyze the scenario where both poorly-endowed and richly-endowed countries coexist. Therefore, we consider a world composed of N_1 poorly-endowed countries and N_2 richly-endowed countries, with $N_1 + N_2 = N$. Assume that the emission technology is the same for both types of countries, $\phi_1 = \phi_2 = 1$. From equation (7) and noting that $\lambda_1(t) = \lambda_2(t) = \lambda(t)$, the following system is obtained.

$$\begin{cases} \left[1 - (1 - \gamma) \frac{N_1 - 1}{N - 1}\right] e_1(t) - (1 - \gamma) \frac{N_2}{N - 1} e_2(t) = U'^{-1}(\lambda(t)) - \pi_1 \\ -(1 - \gamma) \frac{N_1}{N - 1} e_1(t) + \left[1 - (1 - \gamma) \frac{N_1 - 1}{N - 1}\right] e_2(t) = U'^{-1}(\lambda(t)) - \pi_2 \end{cases} \quad (17)$$

Solving the system (17) gives the equilibrium emissions of both countries as follows.

$$\begin{cases} e_1(\lambda(t), \pi_1, \pi_2, N_1, N_2) = \frac{[U'^{-1}(\lambda(t)) - \pi_1] \left[1 - (1 - \gamma) \frac{N_2 - 1}{N - 1}\right] + (1 - \gamma) \frac{N_2}{N - 1} [U'^{-1}(\lambda(t)) - \pi_2]}{D} \\ e_2(\lambda(t), \pi_1, \pi_2, N_1, N_2) = \frac{[U'^{-1}(\lambda(t)) - \pi_2] \left[1 - (1 - \gamma) \frac{N_1 - 1}{N - 1}\right] + (1 - \gamma) \frac{N_1}{N - 1} [U'^{-1}(\lambda(t)) - \pi_1]}{D} \end{cases} \quad (18)$$

where

$$\begin{aligned} D &= \left[1 - (1 - \gamma) \frac{N_1 - 1}{N - 1}\right] \left[1 - (1 - \gamma) \frac{N_2 - 1}{N - 1}\right] - \left(\frac{1 - \gamma}{N - 1}\right)^2 N_1 N_2 \\ &= \gamma + \frac{1 - \gamma}{N - 1} > 0 \end{aligned} \quad (19)$$

From equations (18), it can be shown that

$$e_1(\lambda(t), \pi_1, \pi_2, N_1, N_2) - e_2(\lambda(t), \pi_1, \pi_2, N_1, N_2) = \frac{\gamma}{D} (\pi_2 - \pi_1) > 0. \quad (20)$$

So, with identical level of emission technology, poorly-endowed countries are more polluting than richly-endowed countries in a asymmetric world composed of both types of countries.

3.2 Mean-preserving spread and path of the pollution stock externality

In this section, we deal with a specific question. How does reducing inequality among richly-endowed countries and poorly-endowed countries affect the path of the pollution stock. Specifically, we start with an initial distribution of endowment, and then consider a redistribution that is more equalitarian than the previous one. We look at how the path of the pollution responds to a more equalitarian redistribution from richly-endowed countries in favor of poorly-endowed countries. Indeed, for a given distribution,¹¹

$$\{(\pi_1, N_1), (\pi_2, N_2)\}, \quad (21)$$

¹¹The couple (π_i, N_i) means that N_i countries are endowed with π_i .

let us define a mean function and the variance function as follows:

$$\begin{cases} E(\pi_1, \pi_2) = \frac{1}{N_1+N_2} [N_1\pi_1 + N_2\pi_2] = \bar{\pi}, \\ V(\pi_1, \pi_2) = \frac{1}{N_1+N_2} [N_1(\pi_1 - \bar{\pi})^2 + N_2(\pi_2 - \bar{\pi})^2]. \end{cases} \quad (22)$$

Assume that at time t the distribution of endowments among richly-endowed countries and poorly-endowed countries is given by

$$\{(\pi_1, N_1), (\pi_2, N_2)\}, \quad (23)$$

Let us find a mean-preserving spread redistribution

$$\{(\pi'_1, N_1), (\pi'_2, N_2)\} \quad (24)$$

of endowments among richly-endowed countries and poorly-endowed countries which satisfies

$$\begin{cases} V(\pi'_1, \pi'_2) = \alpha V(\pi_1, \pi_2), \\ E(\pi'_1, \pi'_2) = E(\pi_1, \pi_2) = \bar{\pi}, \end{cases} \quad (25)$$

where $0 < \alpha < 1$. The parameter α captures the degree of inequality reduction among richly-endowed countries and poorly-endowed countries.¹² Solving the system of equations (25) leads to the following system of solutions:

$$\begin{cases} \pi'_1(\alpha) = \bar{\pi} + \sqrt{\alpha \frac{N_2}{N_1} V(\pi_1, \pi_2)} \\ \pi'_2(\alpha) = \bar{\pi} - \sqrt{\alpha \frac{N_1}{N_2} V(\pi_1, \pi_2)} \end{cases} \quad (26)$$

or

$$\begin{cases} \pi'_1(\alpha) = \bar{\pi} - \sqrt{\alpha \frac{N_2}{N_1} V(\pi_1, \pi_2)} \\ \pi'_2(\alpha) = \bar{\pi} + \sqrt{\alpha \frac{N_1}{N_2} V(\pi_1, \pi_2)} \end{cases} \quad (27)$$

To find the convenient solution, let us impose the following validation conditions.

$$\begin{cases} \pi'_1(\alpha) \geq 0, \\ \pi'_2(\alpha) \geq 0, \\ \pi_1 < \pi'_1(\alpha), \\ \pi'_2(\alpha) < \pi_2. \end{cases} \quad (28)$$

¹²The parameter α can be thought of as a Gini-type inequality index, where 0 indicates perfect equality and 1 indicates maximum inequality.

It could be easily shown that only the system of redistribution (27) satisfies the validation conditions. From (27), we have that

$$\pi_1'(\alpha) - \pi_1 = \frac{N_2}{N_1 + N_2}(1 - \sqrt{\alpha})(\pi_2 - \pi_1) > 0. \quad (29)$$

and

$$\pi_2'(\alpha) - \pi_2 = \frac{N_1}{N_1 + N_2}(1 - \sqrt{\alpha})(\pi_1 - \pi_2) < 0. \quad (30)$$

What is the impact of an α -redistribution (given by (27)) that starts at time t from richly-endowed countries in favor of poorly-endowed countries on the pace of accumulation of the pollution stock ? Given a starting pollution stock $S(t)$, the pace of accumulation of the pollution stock over the interval $[t, t + dt]$ under the status quo, "business-as-usual," is given by

$$\dot{S}(\lambda(t), \pi_1, \pi_2) = E(\lambda(t), \pi_1, \pi_2) - kS(t) > 0, \quad (31)$$

where $E(\lambda(t), \pi_1, \pi_2) = N_1 e_1(\lambda(t), \pi_1, \pi_2) + N_2 e_2(\lambda(t), \pi_1, \pi_2)$.

With an α -redistribution from the richly-endowed in favor of the poorly-endowed, the pace of accumulation of the pollution stock over the interval $[t, t + dt]$ is given by

$$\dot{S}(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) = E(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) - kS(t), \quad (32)$$

where $E(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) = N_1 e_1(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) + N_2 e_2(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha))$.

We can show that, from equations (31) and (32), the change in the pace of the accumulation of the pollution stock can be written

$$\begin{aligned} \dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) - \dot{S}(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) &= E(\lambda(t), \pi_1, \pi_2, N_1, N_2) - E(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)), \\ &= \frac{1}{D} \left(\frac{N - \gamma}{N - 1} \right) N_1 (\pi_1'(\alpha) - \pi_1) + N_2 (\pi_2(\alpha)' - \pi_1). \end{aligned} \quad (33)$$

Pursuing to equations (33), (29) and (30) it follows that

$$\begin{aligned} \dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) - \dot{S}(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) &= \frac{1}{D} \left(\frac{N - \gamma}{N - 1} \right) N_1 N_2 (1 - \sqrt{\alpha})(\pi_2 - \pi_1 + \pi_1 - \pi_2). \\ &= 0. \end{aligned} \quad (34)$$

For any given starting pollution stock $S(t)$, let us recall that the pace of the pollution when an α -redistribution is implemented over the time interval $[t, t + dt]$ is given by

$$\dot{S}(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) = \frac{S_\alpha(t + dt) - S(t)}{dt} \quad (35)$$

where $S_\alpha(t + dt)$ is the pollution stock at time $t + dt$.

The pace of the pollution under the status quo, "business-as-usual," is given by

$$\dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) = \frac{S_1(t + dt) - S(t)}{dt} \quad (36)$$

where $S_1(t + dt)$ is the pollution stock at time $t + dt$.

From equations (35) and (36), it follows that

$$\dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) - \dot{S}(\lambda(t), \pi'_1(\alpha), \pi'_2(\alpha)) = \frac{S_1(t + dt) - S_\alpha(t + dt)}{dt} \quad (37)$$

Therefore, combining equations (37) and (34) leads to

$$S_1(t + dt) = S_\alpha(t + dt) \quad (38)$$

Equation (38) tells us that an α -redistribution from richly-endowed countries in favor of poorly-endowed countries will not change the path of the pollution stock over the time interval $[t, t + dt]$. Indeed, the two types of countries have the same emission technology (so the same capacity to reduce emission, all things being equal elsewhere). Therefore, after a mean-preserving redistribution in favor of the poorly-endowed countries, these countries decrease their emission amount. However, this decrease compensates for the increase in emission on the richly-endowed countries side.

3.3 Redistribution and path of the pollution stock: the case of differences in emission technology

Now let us assume that countries differ also by the emission technology, that is $\phi_1 \neq \phi_2$. Then the system of equations for the equilibrium emissions is obtained by modifying (17) as follows:

$$\begin{cases} \phi_1 \left[1 - (1 - \gamma) \frac{N_1 - 1}{N - 1} \right] e_1(t) - \phi_2 (1 - \gamma) \frac{N_2}{N - 1} e_2(t) = U'^{-1}(\lambda(t)) - \pi_1 \\ -\phi_1 (1 - \gamma) \frac{N_1}{N - 1} e_1(t) + \phi_2 \left[1 - (1 - \gamma) \frac{N_1 - 1}{N - 1} \right] e_2(t) = U'^{-1}(\lambda(t)) - \pi_2 \end{cases} \quad (39)$$

Solving the system (39), and as we have done in Section 3.2, the difference between the pace of accumulation of the pollution stock under the two scenarios - status quo and α -redistribution - over the time interval $[t, t + dt]$ is given by

$$\dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) - \dot{S}(\lambda(t), \pi'_1(\alpha), \pi'_2(\alpha)) = (\pi_2 - \pi_1) \frac{N_1 N_2 \gamma (1 + \sqrt{\alpha})}{\gamma + \frac{1 - \gamma}{N - 1}} \left[\frac{1}{\phi_1} - \frac{1}{\phi_2} \right] \quad (40)$$

For any given starting pollution stock $S(t)$, let us recall that

$$\dot{S}(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) = \frac{S_\alpha(t+dt) - S(t)}{dt} \quad (41)$$

where $S_\alpha(t+dt)$ is the pollution stock at time $t+dt$, given that an α -redistribution is implemented over the time interval $[t, t+dt]$ and

$$\dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) = \frac{S_1(t+dt) - S(t)}{dt} \quad (42)$$

where $S_1(t+dt)$ is the pollution stock at time $t+dt$, under the status quo, "business-as-usual," over the time interval $[t, t+dt]$.

From equations (41) and (42), it follows that

$$\dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) - \dot{S}(\lambda(t), \pi_1'(\alpha), \pi_2'(\alpha)) = \frac{S_1(t+dt) - S_\alpha(t+dt)}{dt} \quad (43)$$

Therefore, combining equations (43) and (40) leads to

$$S_1(t+dt) - S_\alpha(t+dt) = \left[(\pi_2 - \pi_1) \frac{N_1 N_2 \gamma (1 - \sqrt{\alpha})}{\gamma + \frac{1-\gamma}{N-1}} \left[\frac{1}{\phi_1} - \frac{1}{\phi_2} \right] \right] dt \quad (44)$$

To summarize, as suggested by the result below, the pollution stock can change in various ways when we reduce inequality among countries in terms of endowments.

$$\begin{cases} S_1(t+dt) < S_\alpha(t+dt) & \text{if and } \phi_2 < \phi_1 \\ S_1(t+dt) = S_\alpha(t+dt) & \text{if and } \phi_2 = \phi_1 \\ S_1(t+dt) > S_\alpha(t+dt) & \text{if and } \phi_2 > \phi_1 \end{cases} \quad (45)$$

In other words, if the more equal redistribution is in favor of the countries that have the best emission technology (higher ϕ_i) then reducing inequality leads to emission decrease. But, if the redistribution is in favor of the countries with low technology then lower inequality leads to higher emission amount.

4 Redistribution and Path of the pollution stock in a more heterogenous world

Our analysis so far has assumed that all countries can be aggregated into two subgroups. Further analysis may be gained by considering a more heterogeneous world composed of four types of countries: countries with technology ϕ_1 and endowment π_1 (type 1.1), countries with technology ϕ_1 and endowment π_2 (type 1.2), countries with technology ϕ_2 and endowment π_1 (type 2.1), and

countries with technology ϕ_2 and endowment π_2 (type 2.2). Let us denote by e_{ij} the emission amount of a country of type $i.j$.¹³

We can adapt equation (7) as follows. For type 1.1 we have

$$\begin{aligned} \phi_1 e_{11}(t) - \frac{1-\gamma}{N-1} ((N_{11}-1)\phi_1 e_{11}(t) + N_{12}\phi_1 e_{12}(t) + N_{21}\phi_2 e_{21}(t) \\ + N_{22}\phi_2 e_{22}(t)) = u'^{-1} \left(\frac{\lambda(t)}{\phi_1} \right) - \pi_1, \end{aligned} \quad (46)$$

where N_{ij} is the number of countries of type $i.j$. Writing (46) for each type gives the following system of four equations with four unknown variables $e_{11}(t)$, $e_{12}(t)$, $e_{21}(t)$, $e_{22}(t)$. That is:

$$\begin{aligned} \phi_1 \left[1 - \frac{1-\gamma}{N-1} (N_{11}-1) \right] e_{11} - \phi_1 \frac{1-\gamma}{N-1} N_{12} e_{12} - \frac{1-\gamma}{N-1} N_{21} \phi_2 e_{21} \\ - \frac{1-\gamma}{N-1} N_{22} \phi_2 e_{22} = u'^{-1} \left(\frac{\lambda}{\phi_1} \right) - \pi_1 \end{aligned} \quad (47)$$

$$\begin{aligned} -\phi_1 \frac{1-\gamma}{N-1} N_{11} e_{11} + \phi_1 \left[1 - \frac{1-\gamma}{N-1} (N_{12}-1) \right] e_{12} - \frac{1-\gamma}{N-1} N_{21} \phi_2 e_{21} \\ - \frac{1-\gamma}{N-1} N_{22} \phi_2 e_{22} = u'^{-1} \left(\frac{\lambda}{\phi_1} \right) - \pi_2 \end{aligned} \quad (48)$$

$$\begin{aligned} -\phi_1 \frac{1-\gamma}{N-1} N_{11} e_{11} - \phi_1 \frac{1-\gamma}{N-1} N_{12} e_{12} + \phi_2 \left[1 - \frac{1-\gamma}{N-1} (N_{21}-1) \right] e_{21} - \\ \frac{1-\gamma}{N-1} N_{22} \phi_2 e_{22} = u'^{-1} \left(\frac{\lambda}{\phi_2} \right) - \pi_1 \end{aligned} \quad (49)$$

$$\begin{aligned} -\phi_1 \frac{1-\gamma}{N-1} N_{11} e_{11} - \phi_1 \frac{1-\gamma}{N-1} N_{12} e_{12} - \phi_2 \frac{1-\gamma}{N-1} N_{21} e_{21} \\ + \phi_2 \left[1 - \frac{1-\gamma}{N-1} (N_{22}-1) \right] e_{22} = u'^{-1} \left(\frac{\lambda}{\phi_2} \right) - \pi_2 \end{aligned} \quad (50)$$

Subtracting (48) from (47) yields

$$\frac{N-\gamma}{N-1} \phi_1 e_{11} - \frac{N-\gamma}{N-1} \phi_1 e_{12} = \pi_2 - \pi_1 \quad (51)$$

And (50) from (49) yields

$$\frac{N-\gamma}{N-1} \phi_2 e_{21} - \frac{N-\gamma}{N-1} \phi_2 e_{22} = \pi_2 - \pi_1 \quad (52)$$

¹³Some intuition can be gained by looking at the ranking of the US states in terms of income per capita and innovation, collected data from Bloomberg. As shown in Table 1 in the Appendix, states that rank high in terms of income per capita are not necessarily the most innovative.

From (51) and (52) we have

$$e_{12}(t) = -\frac{N-1}{\phi_1(N-\gamma)}(\pi_2 - \pi_1) + e_{11}(t) \quad (53)$$

$$\text{and } e_{21}(t) = \frac{N-1}{\phi_2(N-\gamma)}(\pi_2 - \pi_1) + e_{22}(t) \quad (54)$$

Plugging (53) and (54) into (47) and (50) gives

$$\begin{aligned} & \phi_1 \left[1 - \frac{1-\gamma}{N-1}(N_{11} - 1) \right] e_{11} - \phi_1 \frac{1-\gamma}{N-1} N_{12} \left[-\frac{N-1}{\phi_1(N-\gamma)}(\pi_2 - \pi_1) + e_{11} \right] \\ & - \frac{1-\gamma}{N-1} N_{21} \phi_2 \left[\frac{N-1}{\phi_2(N-\gamma)}(\pi_2 - \pi_1) + e_{22} \right] - \frac{1-\gamma}{N-1} N_{22} \phi_2 e_{22} = u'^{-1} \left(\frac{\lambda}{\phi_1} \right) - \pi_1 \end{aligned} \quad (55)$$

and

$$\begin{aligned} & -\phi_1 \frac{1-\gamma}{N-1} N_{11} e_{11} - \phi_1 \frac{1-\gamma}{N-1} N_{12} \left[-\frac{N-1}{\phi_1(N-\gamma)}(\pi_2 - \pi_1) + e_{11} \right] \\ & -\phi_2 \frac{1-\gamma}{N-1} N_{21} \left[\frac{N-1}{\phi_2(N-\gamma)}(\pi_2 - \pi_1) + e_{22} \right] + \phi_2 \left[1 - \frac{1-\gamma}{N-1}(N_{22} - 1) \right] e_{22} = u'^{-1} \left(\frac{\lambda}{\phi_2} \right) - \pi_2 \end{aligned} \quad (56)$$

By solving for e_{11} and e_{22} the system given by (55) and (56) we have

$$\begin{aligned} e_{11}(t) = & \frac{1}{\phi_1 \gamma (N-\gamma)} \left\{ [N-\gamma - (1-\gamma)(N_{21} + N_{22})] \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_1} \right) - \pi_1 \right] \right. \\ & \left. + (1-\gamma)(N_{21} + N_{22}) \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_2} \right) - \pi_2 \right] + (1-\gamma)(N_{21} - N_{12})(\pi_2 - \pi_1) \right\} \end{aligned} \quad (57)$$

and

$$\begin{aligned} e_{22}(t) = & \frac{1}{\phi_2 \gamma (N-\gamma)} \left\{ [N-\gamma - (1-\gamma)(N_{11} + N_{12})] \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_2} \right) - \pi_2 \right] \right. \\ & \left. + (1-\gamma)(N_{11} + N_{12}) \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_1} \right) - \pi_1 \right] + (1-\gamma)(N_{21} - N_{12})(\pi_2 - \pi_1) \right\} \end{aligned} \quad (58)$$

Using (57) and (58) to rewrite (53) and (54) we get to

$$\begin{aligned} e_{12}(t) = & \frac{1}{\phi_1 \gamma (N-\gamma)} \left\{ [N-\gamma - (1-\gamma)(N_{21} + N_{22})] \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_1} \right) - \pi_1 \right] \right. \\ & \left. + (1-\gamma)(N_{21} + N_{22}) \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_2} \right) - \pi_2 \right] + [(1-\gamma)(N_{21} - N_{12}) - \gamma(N-1)](\pi_2 - \pi_1) \right\} \end{aligned} \quad (59)$$

and

$$\begin{aligned} e_{21}(t) = & \frac{1}{\phi_2 \gamma (N-\gamma)} \left\{ [N-\gamma - (1-\gamma)(N_{11} + N_{12})] \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_2} \right) - \pi_2 \right] \right. \\ & \left. + (1-\gamma)(N_{11} + N_{12}) \left[u'^{-1} \left(\frac{\lambda(t)}{\phi_1} \right) - \pi_1 \right] + [(1-\gamma)(N_{21} - N_{12}) + \gamma(N-1)] \right\} \end{aligned} \quad (60)$$

The total emission is $E(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) = N_{11}e_{11} + N_{12}e_{12} + N_{21}e_{21} + N_{22}e_{22}$. Let us consider another redistribution (π'_1, π'_2) of endowments, as we did before, i.e.

$$\pi'_1(\alpha) = \bar{\pi} - \frac{N_{12} + N_{22}}{N} (\pi_2 - \pi_1) \sqrt{\alpha} \quad (61)$$

$$\pi'_2(\alpha) = \bar{\pi} + \frac{N_{11} + N_{21}}{N} (\pi_2 - \pi_1) \sqrt{\alpha}, \quad (62)$$

where $\bar{\pi} = \frac{(N_{11} + N_{21})\pi_1 + (N_{12} + N_{22})\pi_2}{N}$.

We denote by $E'(\lambda(t), \pi_1(\alpha), \pi_2(\alpha))$ the total emissions under the endowments path (π'_1, π'_2) . As in the previous section, the change in the pace of the accumulation of the pollution stock between the two scenarios can be written as

$$\dot{S}(\lambda(t), \pi_1(\alpha), \pi_2(\alpha)) - \dot{S}(\lambda(t), \pi'_1(\alpha), \pi'_2(\alpha)) = E(\lambda(t), \pi_1, \pi_2, N_1, N_2) - E(\lambda(t), \pi'_1(\alpha), \pi'_2(\alpha))$$

Therefore, it can be shown that the difference in pollution stock between the two scenarios is given by

$$S_1(t + dt) - S_\alpha(t + dt) = \left[\frac{(N-1)(1-\sqrt{\alpha})(\pi_2 - \pi_1)}{N(N-\gamma)} (N_{11}N_{22} - N_{12}N_{21}) \left(\frac{1}{\phi_1} - \frac{1}{\phi_2} \right) \right] dt. \quad (63)$$

Compared with what we have seen in Section 3.3, the path of the pollution stock also depends not only on differences in emission technologies, but also on the sign of the expression $N_{11}N_{22} - N_{12}N_{21}$ that captures the role played by the relative size of different types of countries. To understand the intuition behind this result, let us consider the subgroup of $N_{12} + N_{22}$ high-technology countries and the subgroup of $N_{11} + N_{21}$ low-technology countries. The path of the pollution stock appear to depend ultimately on which sub group of countries is better off with the α -redistributive policy. In other words, we can focus our attention on the average amount $\bar{\pi}_{\phi_1}(\alpha)$ of the non polluting good each for the subgroup of countries with technology level ϕ_1 , and the average amount $\bar{\pi}_{\phi_2}(\alpha)$ of the non polluting good for the subgroup of countries with technology ϕ_2 . The average amount $\bar{\pi}_{\phi_1}(\alpha)$ is equal to $\frac{N_{11}\pi_1(\alpha + N_{12}\pi_2(\alpha))}{N_{11} + N_{12}}$, and the average amount $\bar{\pi}_{\phi_2}(\alpha)$ is equal to $\frac{N_{21}\pi_1(\alpha + N_{22}\pi_2(\alpha))}{N_{21} + N_{22}}$. If $N_{11}N_{22} - N_{12}N_{21}$ is positive, then $\bar{\pi}_{\phi_2}(\alpha)$ is greater than $\bar{\pi}_{\phi_1}(\alpha)$. In other words, an α -redistribution from the richly-endowed countries in favor of the poorly-endowed countries will mitigate the stock of pollution if it provides in average more non polluting good to the subgroup of high-technology countries.

5 Concluding remarks

This paper has built a dynamic game model of pollution control where countries are differentiated by a non polluting endowment. In addition to the non polluting good, each country produces and consumes another polluting good using a given technology. We have shown that a more equal redistribution from richly-endowed countries in favor of poorly endowed countries may lead to a greater stock of pollution. If the redistribution is in favor of the countries that have the best emission technology then reducing inequality

leads to emission decrease. But, if the redistribution is in favor of the countries with lower technology then lower inequality leads to higher emission amount.

The problem of climate change, also known as the ultimate commons problem of the twenty-first century (Stavins, 2011), raises multi-dimensional complex issues. Warsaw climate talks set 2015 target for plans to mitigate climate change. Our paper provides a framework that may be prove useful for informing the political economy of redistribution related to climate change mitigation.

The results of this paper provide a basis for future work for incorporating institutional choices design in analyzing transboundary pollution control. In this way, it would be interesting to analyze a two-stage game where in first stage countries choose their constitutional design and in a second step they choose their emission strategies.

Appendix

Table 1: Some intuition from the ranking of sates in US (source: Bloomberg)

Richly-endow/high-tech states	Richly-endow/lo-tech states	Poorly-endow/high-tech states
California: 10th inco, 2nd innov	Alaska: 2nd inco, 26th innov	Oregon: 29th Inco, 5th innov
Washington: 12th inco, 1st innov	Hawai: 8nd inco, 31th innov	Arizona: 30th Inco, 14th innov
Maryland: 1st inco, 9th innov	Delaware: 9th inco, 28th innov	North Carolina: 39th inco, 15th innov
Connecticut: 4th inco, 4th innov,	North Dakota: 20th inco, 37th innov	
New Jersey: 3th inco, 7th innov	Alaska: 2nd income, 26th innov	

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